Frustrated Total Internal Reflection with Microwaves

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Abstract

We look at the widely studied phenomenon of frustrated total internal reflection of microwaves within two wax prisms. We compare the reflected microwave measured by a detector to the theory found in Zhu *et al* (1986)[1]. In order to utilize the theory, properties of the system such as the index of refraction of the wax prisms and the wavelength, are measured.

We measure the reflected wave and compare to the expected trend as a function of prism separation. In addition we observe some interesting oscillations in the electric field when the separation of prisms is greatest.

Introduction

Total Internal Reflection (TIR) occurs when a wave is completely reflected instead of transmitting any of the wave through a boundary to a medium of lower index of refraction. This only happens if the angle of incidence is greater than a critical angle determined by the indices of refraction. In the case of TIR, solutions to Maxwell's equations include a non-zero electromagnetic field outside the reflecting boundary between the first and second media. These fields are not sinusoidal and therefore carry no energy, however if a third medium is introduced causing the second to act as a thin film on the order of the wavelength, a phenomenon known as frustrated total internal reflection (FTIR) occurs. In FTIR, some energy of the wave is carried across the classically forbidden region of second medium into the third. This is similar to the way quantum particles can tunnel through barriers.

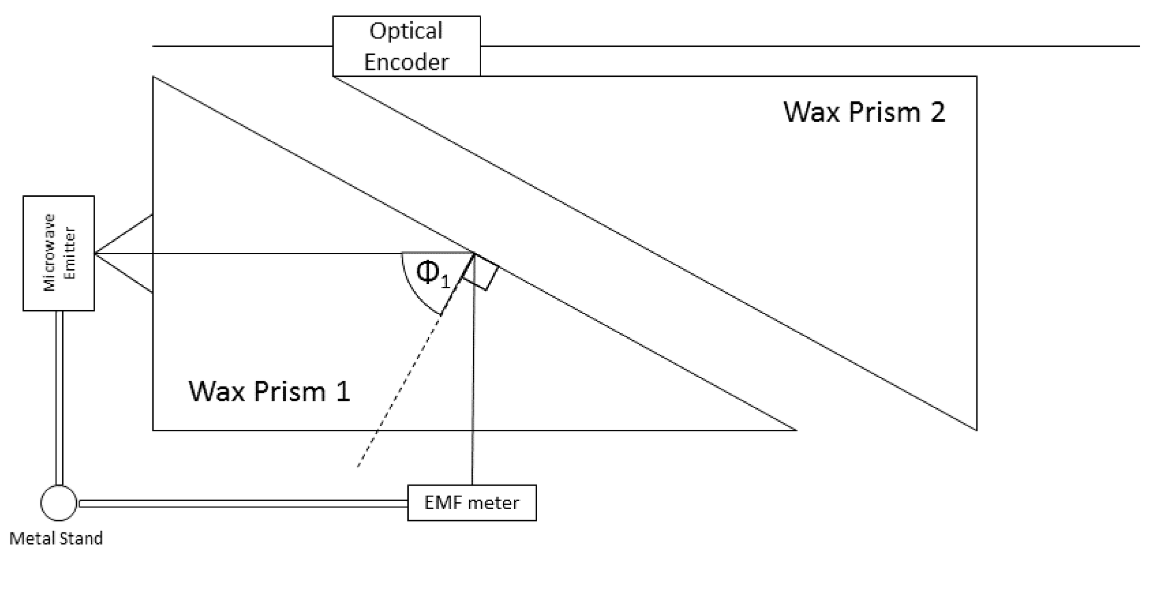
We attempt to measure this phenomenon with wax prisms and microwaves. We choose to use microwaves because FTIR requires the gap between prisms to be on the order of the wavelength. Microwaves provide the dimensions required to produce FTIR while still being on a scale where specialized instruments are not required.

Methods

We utilize a Gunn diode microwave emitter attached to a horn antenna. The horn antenna is placed flush to the short end of one wax prism so that the emitted wave is polarized in the TM mode. The prisms are cut from the same rectangular block to have angles of 30, 60, and 90 degrees. The wave travels through the short side to the hypotenuse such that it has an angle of incidence of 60 degrees. A diode EMF (Electro Motive Force) meter is placed along the long side to measure the magnitude of the electric field reflected via TIR from the wax to air boundary of the hypotenuse. The readout of the EMF meter is displayed on a digital voltmeter.

A second wax prism is oriented such that when the hypotenuses of the prisms are pressed together, a rectangular prism is formed. The second prism sits upon a table suspended on rollers so it can move along one axis varying the separation between the hypotenuse faces. An optical encoder attached to the table measures the position of the table along it's axis of motion and provides a digital readout. To perform the experiment we gradually increase the separation of the wax prisms recording both the separation and the reflected electric field magnitude.

In order to make theoretical predictions we require measurements of the wavelength of the microwaves as well as the index of refraction of the wax. We measure both the refractive index and the wavelength by creating a standing wave between the horn and a sheet of aluminum. The EMF meter is used to determine the locations of nodes on the standing wave. We also measure the refractive index by mapping the angular dependence of the electric field as it refracts through the wax prism.



Theory

Beginning with the Fresnel equations we can derive an relation for the electric field magnitude of a reflected wave in the TM mode[1]. In the case of total internal reflection, assuming that the magnetic properties of all media are uniform, and the media are non-conducting, the first formula below for the reflection coefficient can be derived. The following equations give definitions of the variables used in terms of the index of refraction (n), the angle of incidence (ϕ_1), the wavelength (λ), and the prism separation (d).

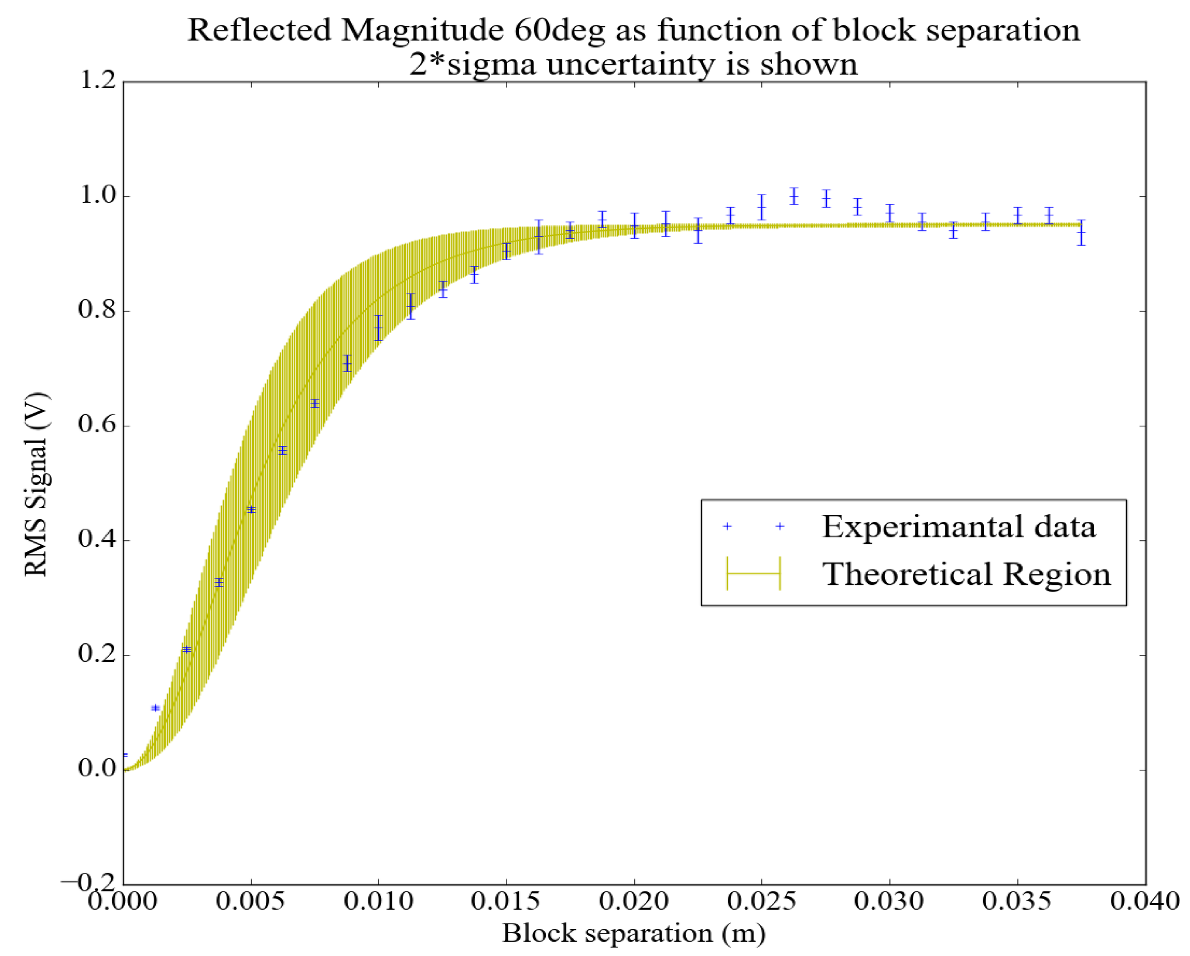
$$R = 1 - T$$

$$T = \frac{1}{\alpha \sinh^2 y + 1}$$

$$\alpha = \frac{(n^2-1)^2((n^2+1)\sin^2\phi_1 - 1)^2}{4n^2\cos^2\phi_1(n^2\sin^2\phi_1 - 1)}$$

$$y = \frac{2\pi d}{\lambda(n^2\sin^2\phi_1 - 1)^{\frac{1}{2}}}$$

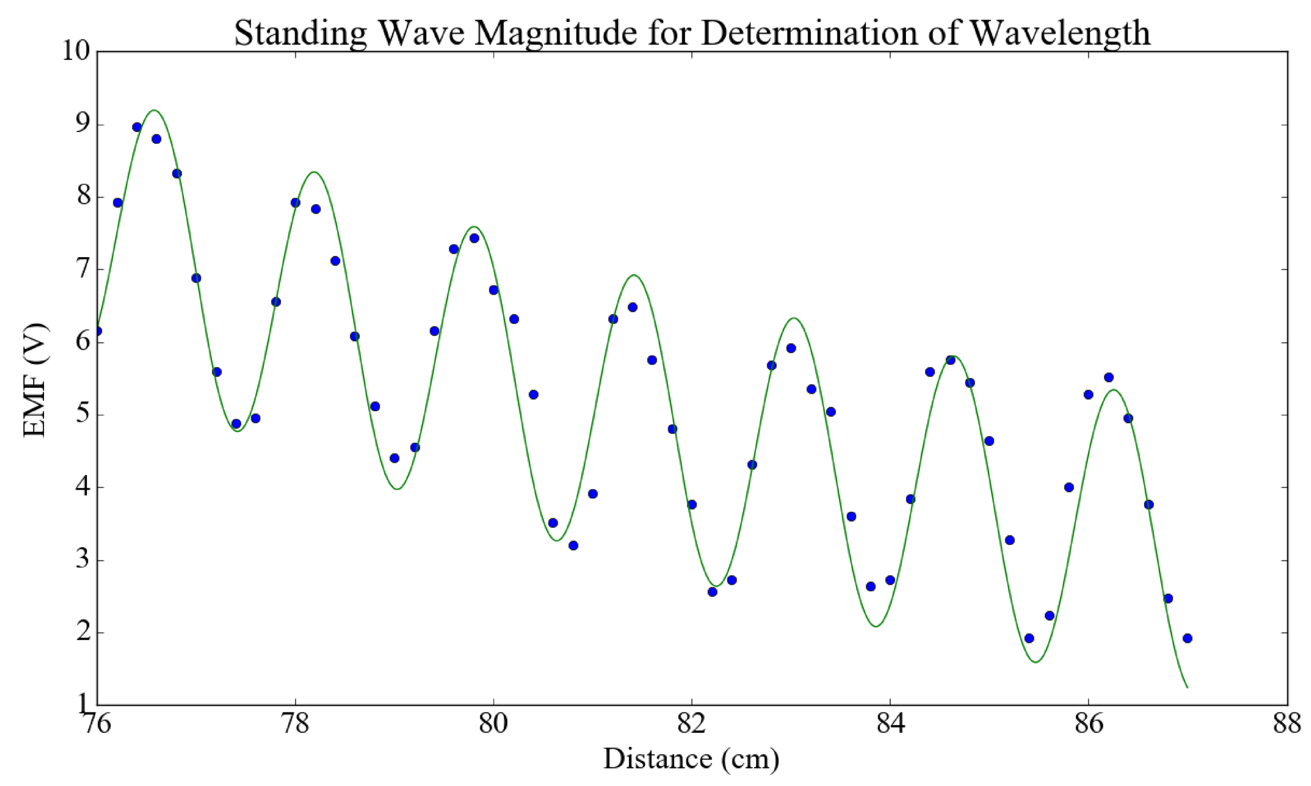
Results



Initially we measured the wavelength of our microwave using the standing wave method described earlier. Since we know the distance between nodes in a standing wave is one half the wavelength, from our measurements we can conclude that the wavelength is 3.222±0.011 cm. This is consistent with the manufacturer's specified frequency of 9 GHz.

We measured the index of refraction using a variety of methods. The method with the smallest uncertainty was measuring refraction through the prism and applying Snell's law. With this method we found the index of refraction to be 1.2±0.3. While this is our best estimate, it does give us a 95% confidence interval of 0.6 to 1.8 which is rather a large range. Because of this, we will leave it as a fit parameter during our comparison with theory.

The magnitude of the reflected electric field as a function of prism separation is shown in the plot above. The data clearly does exhibit a trend consistent with less of the wave being able to transmit across the gap as separation increases. When comparing this to the theoretical values predicted by the equations specified above we find that the uncertainty range of the experimental data and theory overlap in the majority of cases. The theory deviates from experimental data both when the separation of the prisms is very small or very large. The theoretical region is marked on the plot above. To generate this, a least squares fit was used to determine a value for the index of refraction because our measurement of this quantity is unreliable. We find a fitted value of the refractive index of 1.44±.001 which is a greater value than we measure via other methods. The uncertainty of this fit is also much lower than that of our direct measurements suggesting that it is more accurate.



Conclusions

When fitting our data, we allow for some loss due to attenuation and the divergence of the beam. Our fit suggests an approximate 5% loss which may be from these factors. This prompts us to investigate the beam profile further in the future as well as to measure both the reflected and transmitted waves simultaneously. This will allow us to safely say what is causing this loss and allow us to relate the transmittance and reflectance more precisely which we should be able to work into our theory.

Our first inquiry into the profile of the beam and the existence of oscillations in the magnitude of the electric field at large separation leads us to believe there may be some interference at the EMF meter due to various parts of the beam reflecting at different angles of incidence inside the wax prism. If some divergence of the beam is occurring, it would contribute to our uncertainty in the angle of incidence. To account for this we currently estimate an uncertainty of 2 degrees in this value.

Future Work

- We notice oscillations in the electric field reflected by the wax blocks when the separation is at it's greatest. In the future we plan to examine if these oscillations are repeatable, and if so we will attempt to formulate a theory for the oscillations and match that to the observed trends.
- We have the least confidence in our measurement of the index of refraction of the wax, therefore we will remeasure the refractive index with multiple methods to reduce uncertainty.
- We also want to investigate if the transmitted and reflected waves sum to the same as the incident wave throughout the experiment.
- We plan to automate the data collection process so that we are able to obtain more accurate and higher resolution data.

Acknowledgements

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References

1. Zhu, S., A. W. Yu, D. Hawley, and R. Roy. "Frustrated Total Internal Reflection: A Demonstration and Review." *American Journal of Physics* 54.7 (1986): 601-07.